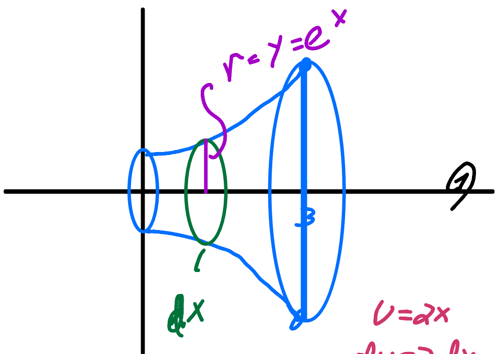
Period 3, March 10, 2025

$y=e^x$, x-axis and $x=3$

around x-axis



$$\int_0^3 \pi r^2\,dx = \int_0^3 \pi y^2\,dx = \pi\int_0^3 (e^x)^2\,dx$$

$$\pi\int_0^3 e^{2x}\,dx = \pi\int e^u \cdot \frac{du}{2} = \frac{\pi}{2}\int e^u\,du$$

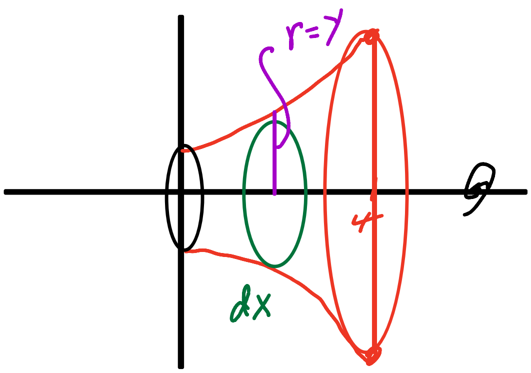
$u=2x$
$du=2dx$
$\frac{du}{2}=dx$

$$\frac{\pi}{2}e^u + c = \frac{\pi}{2}e^{2x}\Big[_0^3$$

$$\frac{\pi}{2}\left[e^{2\cdot 3}-e^{2\cdot 0}\right] = \frac{\pi}{2}(e^6-e^0) = \frac{\pi}{2}(e^6-1)$$

$y=e^x$, x-axis and $x=4$

around x-axis



$$\int_0^4 \pi r^2\,dx = \int_0^4 \pi y^2\,dx = \int_0^4 \pi(e^x)^2\,dx$$

$$\frac{\pi}{2}e^{2x}\Big[_0^4 = \frac{\pi}{2}\left[e^{2\cdot 4}-e^{0\cdot 2}\right]$$

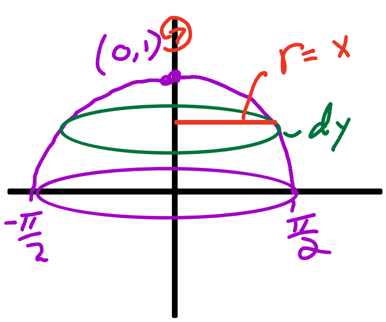
$$\frac{\pi}{2}(e^8-e^0)$$

$$\frac{\pi}{2}(e^8-1) = \frac{e^8\pi}{2}-\frac{\pi}{2}$$

$y=\cos x$

$\arccos y = \arccos(\cos x)$

$\arccos y = \cos^{-1} y = x$



$y=\cos x$, I Quad, around y-axis

$$\int_0^1 \pi r^2\,dy = \int_0^1 \pi x^2\,dy$$

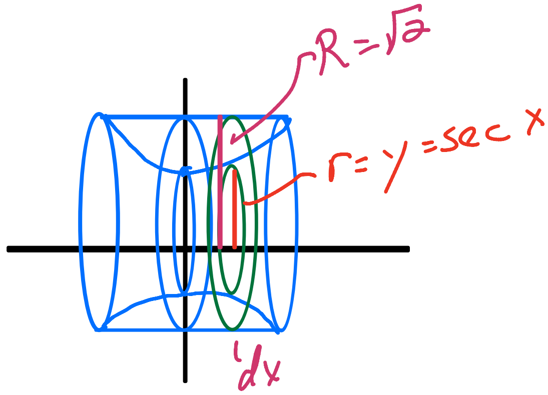
$$\int_0^1 \pi(\arccos y)^2\,dy = 3.5864$$

or

1.14159π

or

$\pi(\pi-2) = \pi^2 - 2\pi$



$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \pi(R^2-r^2)\,dx$$

$$\pi\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} [(\sqrt{2})^2-(\sec x)^2]\,dx$$

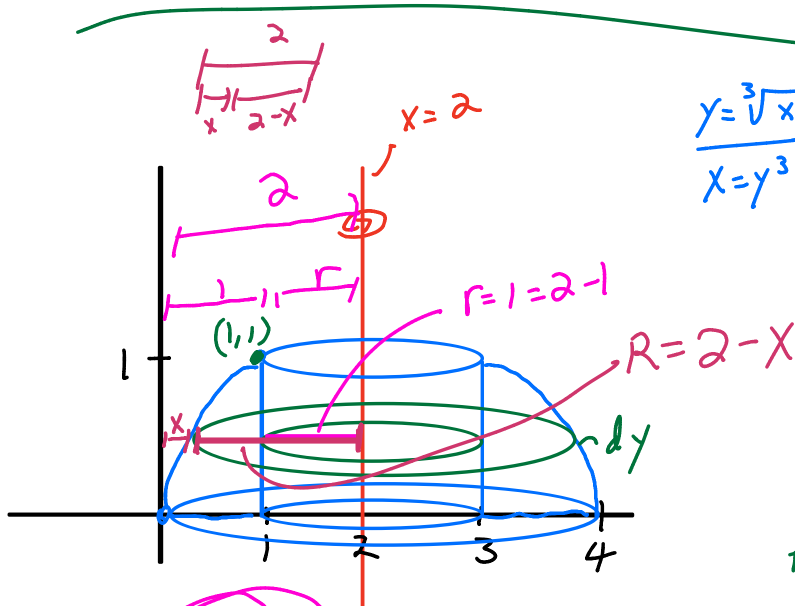
$$\pi\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} [2-\sec^2 x]\,dx$$

$$\pi\left[2x-\tan x\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\pi$$

$$\pi\left[2\left(\frac{\pi}{4}\right)-\tan\left(\frac{\pi}{4}\right)-\left[2\left(-\frac{\pi}{4}\right)-\tan\left(-\frac{\pi}{4}\right)\right]\right]\pi \qquad -(-1)$$

$$\pi\left(\frac{\pi}{2}-1+\frac{\pi}{2}-1\right)=\left(\frac{2\pi}{2}-2\right)\cdot\pi$$

$$(\pi-2)\pi=\pi^2-2\pi$$



$(2-y^3)(2-y^3)=4-2y^3-2y^3+y^6$

$y=\sqrt[3]{x}$, $x=1$, x-axis

$x=y^3$

$$\int_0^1 \pi(R^2-r^2)\,dy$$

$$\pi\int_0^1 [(2-x)^2-1^2]\,dy$$

$$\pi\int_0^1 [(2-y^3)^2-1]\,dy$$

$$\pi\int_0^1 [4-4y^3+y^6-1]\,dy$$

$$\pi\int_0^1 [y^6-4y^3+3]\,dy=\frac{1}{7}y^7-y^4+3y\Big|_0^1$$

$$\pi\left[\tfrac{1}{7}(1)^7 - 1^4 + 3(1)\right] - \left[\cancel{\tfrac{1}{7}(0)^7 - 0^4 + 3(0)}\right]$$

$$\pi\left(\tfrac{1}{7} - 1 + 3 = 2 + \tfrac{1}{7}\right) 2\tfrac{1}{7} \cdot \pi = \tfrac{15}{7} \cdot \pi$$

Given below is a table of function values of $h(x)$. Approximate each of the following definite integrals using the indicated Riemann or Trapezoidal sum, using the indicated subintervals of equal length.

x	-3	-1	1	3	5	7	9
$h(x)$	5	2	-3	-7	-2	6	11

17. $\int_{-3}^{9} h(x)dx$ using three subintervals and a Midpoint Riemann sum.

18. $\int_{-3}^{3} h(x)dx$ using three subintervals and a Trapezoidal sum.

$\frac{1}{2}(b_1+b_2)h$

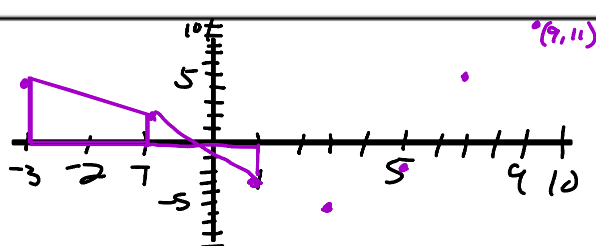
$\cancel{\tfrac{1}{2}}(5+2)\cdot\cancel{2} + \cancel{\tfrac{1}{2}}(2+-3)\cdot\cancel{2} + \cancel{\tfrac{1}{2}}(-3+-7)\cdot\cancel{2}$

$\underbrace{5+2+2}_{9}\ \underbrace{-3-3-7}_{-13} = -4$

19. $\int_{-3}^{9} h(x)dx$ using six subintervals and a Trapezoidal sum.

$-4 + \cancel{\tfrac{1}{2}}(-7+-2)\cancel{2} + \cancel{\tfrac{1}{2}}(-2+6)\cancel{2} + \cancel{\tfrac{1}{2}}(6+11)\cdot\cancel{2}$

$\underbrace{-4-7-2-2}_{-15} + \underbrace{6+6+11}_{+23} = 8$



For questions 20 and 21, approximate the definite integrals. Make a table of values showing your intervals that you used.

20. Approximate $\int_0^{\pi} (2x\sin x)dx$ using four subintervals of equal length and a Right Hand Riemann sum.

$0(\frac{\pi}{4}) + \frac{\pi\sqrt{2}}{4}\cdot\frac{\pi}{4} + \pi\cdot\frac{\pi}{4} + \frac{6\pi\sqrt{2}}{8}\cdot\frac{\pi}{4}$

$0 \to \pi$

$\frac{\pi}{4}$ = subinterval

x	$y = 2x\sin x$
0	$2\cdot 0\cdot\sin 0 = 0$
$\frac{\pi}{4}$	$2\cdot\frac{\pi}{4}\cdot\sin\frac{\pi}{4} = \frac{\pi}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\pi\sqrt{2}}{4}$
$\frac{\pi}{2} = \frac{2\pi}{4}$	$2\cdot\frac{\pi}{2}\sin\frac{\pi}{2} = \pi\cdot\sin\frac{\pi}{2} = \pi\cdot 1 \to \pi$
$\frac{3\pi}{4}$	$2\cdot\frac{3\pi}{4}\cdot\sin\frac{3\pi}{4} = \frac{6\pi}{4}\cdot\frac{\sqrt{2}}{2} = \frac{6\pi\sqrt{2}}{8}$
π	0

21. Approximate $\int_{-2}^{10} (e^2x^2)dx$ using four subintervals of equal length and a Trapezoidal sum.

$\frac{1}{2}(b_1+b_2)h$

$\frac{1}{2}(b_1+b_2)3$

$\frac{12}{4} = 3$

x	y
-2	$4e^2 = e^2(-2)^2$
1	$e^2 = e^2(1)^2$
4	$16e^2 = e^2(4)^2$
7	$49e^2 = e^2(7)^2$
10	$100e^2 = e^2(10)^2$

(3 between each pair of consecutive x-values)

$\frac{1}{2}(4e^2+e^2)\cdot 3 + \frac{1}{2}(e^2+16e^2)\cdot 3 + \frac{1}{2}(16e^2+49e^2)\cdot 3$

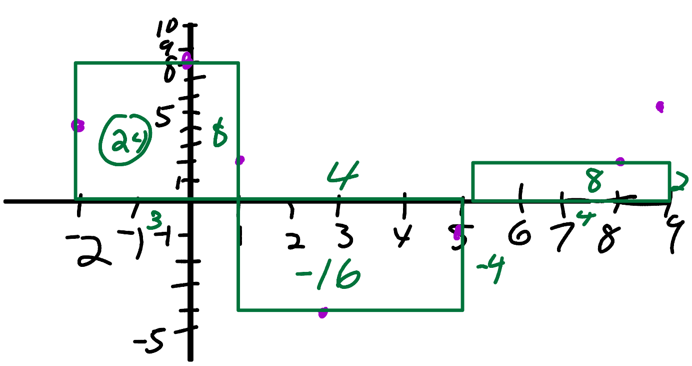
$+ \frac{1}{2}(49e^2+100e^2)\cdot 3$

$\frac{3e^2}{2}[4+1+1+16+16+49+49+100] = \frac{3e^2}{2}(236)$

22. Given the table to the right, approximate $\int_{-2}^{9} P(x)dx$ using three subintervals and a Midpoint Riemann sum.

x	-2	0	1	3	5	8	9
$P(x)$	5	8	2	-4	-1	2	5

$24 + -16 + 8 = 16$



23. Given the table to the right, approximate $\int_{-2}^{9} P(x)\,dx$ using six subintervals and a Trapezoidal sum.

2 1 2

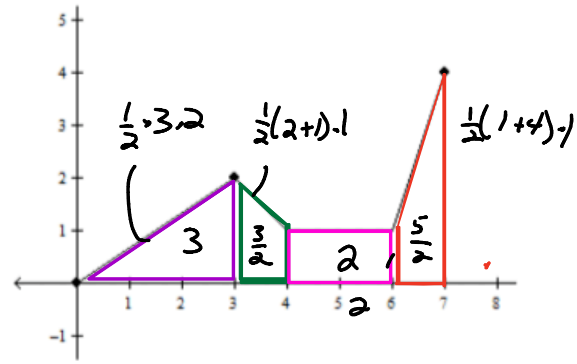
x	-2	0	1	3	5	8	9
$P(x)$	5	8	2	-4	-1	2	5

$\frac{1}{2}(5+8)\cdot 2+\frac{1}{2}(8+2)\cdot 1+\frac{1}{2}(2+{}^{-}4)2+\frac{1}{2}(-4+-1)\cdot 2+\frac{1}{2}(-1+2)\cdot 3$

$+\frac{1}{2}(2+5)\cdot 1$

Pictured below is the graph of $f'(x)$, the first derivative of a function $f(x)$. Use the graph to answer the following questions 38 – 40.

Graph of $f'(x)$



38. What is the value of $\int_0^7 f'(x)dx$

Area under curve

39. If $f(0) = -3$, what is the value of $f(3)$?

$$\text{START} + \int_0^3 F'(x) = 3$$

$$-3+3=0$$

40. If $f(3) = -1$, what is the value of $f(7)$?

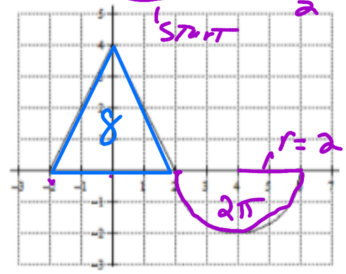
$$-1 + \int_3^7 F'(x) = -1 + \frac{3}{2} + 2 + \frac{5}{2}$$

$$\frac{8}{2} + 2 - 1$$

$$4 + 2 - 1 = 5$$

The graph of $f'(x)$, the derivative of a function, $f(x)$, is pictured below on the interval [–2, 6]. Write and find the value of a definite integral to find each of the indicated values of $f(x)$ below. Also, $f(-2) = 5$.

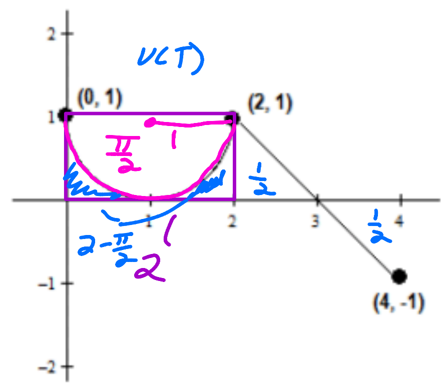
$\frac{\pi r^2}{2}$



Graph of f '

41. Find the value of $f(0)$.	42. Find the value of $f(6)$.
$5 + \int_{-2}^{0} F'(x)\,dx$ START Change from –2 to 0 $5 + 4 = 9$	$5 + \int_{-2}^{6} F'(x)\,dx = 5 + 8 - 2\pi$ $9 + \int_{0}^{6} F'(x)\,dx = 9 + 4 - 2\pi$ $= 13 - 2\pi$

Pictured to the right is the graph of a function which represents a particle's velocity on the interval [0, 4]. Answer the following questions.



48. For what values is the particle moving to the right? Justify your answer.

(0, 3) V(T) is positive

49. For what values is the particle moving to the left? Justify your answer.

(3, 4)

V(T) is negative

52. What is the net distance that the particle travels on the interval [0, 4]?

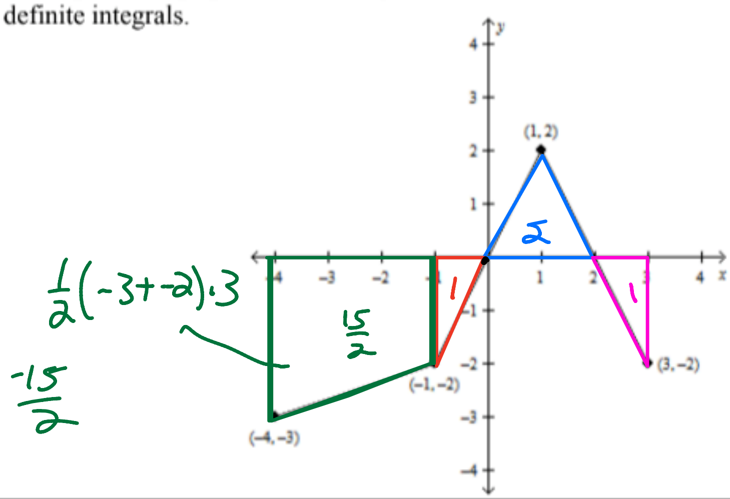
displacement

$2-\frac{\pi}{2}+\cancel{\frac{1}{2}-\frac{1}{2}}=2-\frac{\pi}{2}$

53. What is the total distance that the particle travels on the interval [0, 4]?

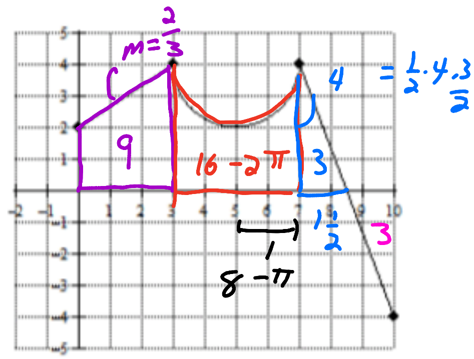
$\int |v(t)|\,dt = 2-\frac{\pi}{2}+\frac{1}{2}+\frac{1}{2}=3-\frac{\pi}{2}$

Pictured to the right is the graph of a function f. In exercises 30 – 35, find the values of each of the following definite integrals.



30. $\int_{-4}^{2} f(x)dx$ $-\frac{15}{2}+-1+2$	31. $\int_{0}^{3} f(x)dx = 2$	32. $\int_{-1}^{1} f(x)dx$
33. $\int_{-4}^{0} f'(x)dx = F(0) - F(-4)$ $0-(-3)=3$	34. $\int_{-1}^{1} f'(x)dx$	35. $\int_{-3}^{2} f'(x)dx$

The graph to the right represents the velocity, $v(t)$ in meters per second, of a particle that is moving along the x – axis on the time interval $0 \le t \le 10$. The initial position of the particle at time $t = 0$ is 12.



43. On what interval(s) of time is the particle moving to the left and to the right? Justify your answer.

$9 = \frac{1}{2}(2+4)\cdot 3$

44. What is the total distance that the particle has traveled on the time interval $0 \le t \le 7$. Leave your answer in terms of π. Indicate units of measure.

$9 + 16 - 2\pi = 25 - 2\pi$

45. What is the net distance that the particle travels on the interval $5 \le t \le 10$? Round your answer to the nearest thousandth. Indicate units of measure.

$8 - \pi + 3 - 3 = 8 - \pi$

46. What is the acceleration of the particle at time $t = 2$? Indicate units of measure.

$V'(2)$ = Slope of $V(T)$ at $x = 2$

$m = \frac{2}{3}$

47. What is the position of the particle at time $t = 5$? Indicate units of measure.

$\text{Start} + \int_0^5 v(T)\,dT = 12 + 9 + 8 - \pi = 29 - \pi$

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity, v, measured in feet per second, and acceleration, a, measured in feet per second per second, are continuous functions. The table below shows selected values of these functions.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

$\frac{4}{5}$

$\frac{4}{15}$ $\quad -\frac{3}{10}$ $\quad -\frac{1}{5}$ $\quad \frac{1}{5}$ $\quad \frac{2}{15}$ $\quad \frac{-2}{10}$

Total distance

54. Using appropriate units, explain the meaning of $\int_0^{60} |v(t)|\,dt$ in terms of the car's motion. Approximate this integral using a midpoint approximation with three subintervals as determined by the table.

$25(-30)+10(-14)+25(0) = 25\cdot 30 + 10\cdot 14 + 0$

$750 + 140 = 890$ FT

55. Using appropriate units, explain the meaning of $\int_{15}^{50} a(t)\,dt$ in terms of the car's motion. Find the exact value of the integral.

Change in velocity From 15 To 50sec

$V(50)-V(15) = 0-(-30) = 30$ FT/sec

56. Is there a value of t such that $a'(t) = 0$? If so, on what interval does such a value exist? Justify your reasoning.

Slope of $a(t)=0$

0 To 30 or 35 To 60
or or
25-35 25 To 60

MVT

57. Using appropriate units, approximate the value of $v'(31)$. What does this value say about the motion of the car at $t = 31$.

Slope of $V(T)$ $\to a(31)$

$dT=1$ $\quad \frac{dV}{dT} = \frac{4}{5} \cdot dT \Rightarrow dV = \frac{4}{5}\cdot 1 = \frac{4}{5}$

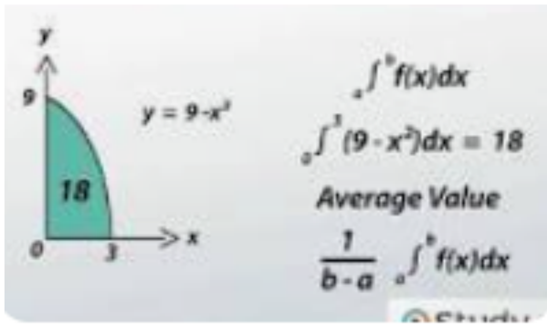
$a(30)=1$

$a(31) \approx 1 + \frac{4}{5} = \frac{9}{5}$

58. Using appropriate units, find the value and explain the meaning of $\frac{1}{35}\int_{25}^{60} a(t)\,dt$.

$\frac{1}{a-b}\int_a^b a(T)\,dT$ → Average Value of $a(T)$ From 60-25

$\frac{1}{35}\left(V(60)-V(25)\right) = \frac{1}{35}\left(10-(-20)\right) = \frac{30}{35}$ FT/sec^2


y
9
$y = 9 - x^2$
18
0
3
x
$\int_a^b f(x)dx$
$\int_0^3 (9 - x^2)dx = 18$
Average Value
$\frac{1}{b-a} \int_a^b f(x)dx$